## A Graphic Method for the Determination of Monomer Reactivity Ratios

The instantaneous copolymer composition equation for a binary components system

$$F_1 = 1 - F_2 = \frac{r_1 f_1^2 + f_1 f_2}{r_1 f_1^2 + 2f_1 f_2 + r_2 f_2^2} \tag{1}$$

has been used for determining monomer reactivity ratios. Equation (1) has been rearranged in the form by Fineman and  $Ross^1$  as follows:

$$\frac{f_1(1-2F_1)}{F_1(1-f_1)} = r_2 + \frac{f_1^2(F_1-1)}{F_1(1-f_1)^2}r_1$$
(2)

The monomer reactivity ratios  $r_i$  are then determined graphically. The  $r_i$  values are estimated from the slope and intercept of the plot of  $f_1(1-2F_1)/F_1(1-f_1)$  against  $f_1^2(F_1-1)/F_1(1-f_1)^2$ , respectively. The disadvantage of this method is that it is sometimes difficult to make a decision regarding the best straight line through a set of points since linear least-squares procedures are not appropriate.<sup>2</sup> In this work, using eq. (1), we propose a new graphic procedure for determining  $r_i$ . In this method, the  $r_i$  values are directly determined from both intercepts of the plot of  $f_1f_2/F_1F_1$  vs.  $f_1$  or  $F_1F_2/f_1f_2$ vs.  $f_1$ . The procedure for making the plot is very simple and the linear least-squares procedure is unnecessary.

Equation (1) is rewritten as

$$\frac{f_1}{F_1} = \frac{r_1 f_1^2 + 2f_1 f_2 + r_2 f_2^2}{r_1 f_1 + f_2} \tag{3}$$

and then its limiting value at  $f_1 = 0$  is found to be

$$\lim_{f_1 \to 0} (f_1/F_1) = r_2 \tag{4}$$

Equation (1) is further rearranged to be

$$F_2 = \frac{f_1 f_2 + r_2 f_2^2}{r_1 f_1^2 + 2f_1 f_2 + r_2 f_2^2}$$

or

$$\frac{f_2}{F_2} = \frac{r_1 f_1^2 + 2f_1 f_2 + r_2 f_2^2}{f_1 + r_2 f_2} \tag{5}$$

The limiting value of  $f_2/F_2$  at  $f_1 = 0$  is also solved:

$$\lim_{f_1 \to 0} (f_2/F_2) = 1 \tag{6}$$

The combination of eqs. (4) and (5) yields

$$\lim_{f_1 \to 0} (f_1/f_2/F_1F_2) = r_2 \tag{7}$$

Similarly, we solve the limiting values of  $f_1/F_1$  and  $f_2/F_2$  at  $f_2 = 0$  or  $f_1 = 1$  for eqs. (3) and (5) and obtain

$$\lim_{f_1 \to 1} (f_1/F_1) = 1 \tag{8}$$

$$\lim_{f_1 \to 1} (f_2/F_2) = r_1 \tag{9}$$

$$\lim_{f_1 \to 1} (f_1 f_2 / F_1 F_2) = r_1 \tag{10}$$

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Data for Determining the Monomer Reactivity Ratios			
System	f_1	$F_1$	
Styrene-methyl methacrylate <sup>3,4</sup>	0.048	0.093	
	0.100	0.171	
	0.111	0.168	
	0.234	0.317	
	0.300	0.371	
	0.478	0.493	
	0.500	0.510	
	0.699	0.647	
	0.700	0.648	
	0.900	0.844	
Methyl methacrylate-isoprene <sup>5</sup>	0.100	0.112	
	0.300	0.261	
	0.500	0.424	
	0.700	0.547	
	0.800	0.576	
	0.850	0.676	
	0.900	0.724	
	0.950	0.843	
	0.957	0.888	
Acrylonitrile–ethyl acrylate <sup>6</sup>	0.380	0.439	
	0.525	0.569	
	0.647	0.697	
	0.741	0.777	
	0.823	0.850	
	0.917	0.930	
Vinyl acetate–methyl methacrylate <sup>7</sup>	0.19	0.01	
	0.37	0.02	
	0.54	0.05	
	0.70	0.08	
	0.86	0.17	
	0.97	0.56	

2748



Fig. 1. Determination of reactivity ratios of monomers for styrene-methyl methacrylate system from intercepts of the plot of  $f_1f_2/F_1F_2$  vs.  $f_1$ : ( $\blacktriangle$ ) Ref. 3; (O) Ref. 4.



Fig. 2. Determination of reactivity ratios of monomers for methyl methacrylate-isoprene system from intercepts of the plot of  $f_1f_2/F_1F_2$  vs.  $f_1$ .



Fig. 3. Determination of reactivity ratios of acrylonitrile-ethyl acrylate system from intercepts of the plot of  $f_1f_2/F_1F_2$  vs.  $f_1$ .

Monomer Reactivity Ratios Obtained by This Method and Literature Values			
Systems	This method (r <sub>1</sub> , r <sub>2</sub> )	Literature values $(r_1, r_2)$	Remarks
Styrene-methyl methacrylate	(0.53, 0.45)	$(0.52 \pm 0.026, 0.46 \pm 0.026)^8$	60°C
Methyl methacrylate-isoprene	(0.22, 0.71)	$(0.22, 0.72)^5$	60°C
Acrylonitrile-ethyl acrylate	(1.18, 0.68)	$(1.17 \pm 0.1, 0.67 \pm 0.02)^9$	50°C
Methyl methacrylate-vinyl acetate	(21.2, 0.016)	$(20 \pm 3, 0.015 \pm 0.015)^{10}$	60°C

TABLE II

According to eqs. (7) and (10), we plot  $f_1f_2/F_1F_2$  against  $f_1$  or  $f_2$ . Then, the monomer reactivity ratios may be found from the intercepts of the curve at  $f_1 = 0$  and  $f_1 = 1$  simultaneously. We do not suggest that the  $r_i$  values be determined by use of eqs. (4) and (9). The main reason is that the plot of  $f_1f_2/F_1F_2$  against  $f_1$  gives a much more smooth curve than the plot of  $f_1/F_1$  against  $f_1$  or  $f_2/F_2$  against



Fig. 4. Determination of reactivity ratios of monomers for vinyl acetate-methyl methacrylate system from intercepts of the plot of  $f_1f_2/F_1F_2$  vs.  $f_1$ .

 $f_1$  does. The other reason is that the former procedure needs only one plot for estimating  $r_i$ , but the latter technique requires two plots for finding  $r_i$ .

This method was verified by testing four copolymerization systems: styrene(1)-methyl methacrylate(2), methyl methacrylate(1)-isoprene(2), acrylonitrile(1)-ethylacrylate(2), and vinyl acetate(1)-methyl methacrylate(2). The data of Bevington et al.,<sup>3</sup> Kuo and Chen,<sup>4</sup> Haward,<sup>5</sup> Brandrup,<sup>6</sup> and Atherton and North<sup>7</sup> shown in Table I, were used to make the plots of Figures 1 to 4. The monomer reactivity ratios  $r_1$  and  $r_2$  for the illustrated four systems were obtained from the intercepts of the plots at  $f_1 = 1$  and 0, respectively. The subindex 1 and 2 denote monomers 1 and 2, respectively. The results were given in Table II. For the MMA-VAc system the monomer reactivity ratios were determined from the logarithmic plot of  $f_1f_2/F_1F_2$  vs.  $f_1$  instead of using the ordinary plot, because the reactivity ratio for vinyl acetate is very small, i.e.,  $r_1 = 0.015$ . If the ordinary plot is used, then the intercept for  $r_1$  is too small for a precision determination. The literature values of these four copolymerizations are also listed in the same table. We find that the monomer reactivity ratios determined by this method are in very good agreement with the literature values.

## References

1. M. Fineman and S. D. Ross, J. Polym. Sci., 5, 269 (1950).

2. R. M. Joshi, J. Macromol. Sci. Chem., A7, 1231 (1973).

3. J. C. Bevington, H. W. Melville, and R. P. Taylor, J. Polym. Sci., XIV, 463 (1954).

4. J. F. Kuo and C. Y. Chen, unpublished data.

5. R. N. Haward, Developments in Polymerization, Vol. 2, Applied Science Publishers, London, 1979, p. 30.

6. J. Brandrup, Faserforsch. Textiltech., 12, 133 (1961).

7. J. N. Atherton and A. M. North, Trans. Faraday Soc., 58, 2049 (1962).

8. F. M. Lewis, C. Walling, W. Cummings, E. R. Briggs, and F. R. Mayo, J. Am. Chem. Soc., 70, 1519 (1948).

9. J. Brandrup, Faserforsch. Textiltech., 12, 135 (1961).

10. F. R. Mayo, C. Walling, F. M. Lewis, and W. F. Hulse, J. Am. Chem. Soc., 70, 1523 (1948).

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